



## Detecting the effects of selection and stochastic forces in archaeological assemblages

P. Jeffrey Brantingham\*, Charles Perreault<sup>1</sup>

Department of Anthropology, University of California, Los Angeles, 341 Haines Hall, Los Angeles, CA 90095, USA

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### ABSTRACT

Detecting and diagnosing the causes of change through time in archaeological assemblages is a core enterprise of archaeology. Evolutionary approaches to this problem typically cast the causes of culture change as being either stochastic in origin, or arising from selection. Stochastic sources of change include random innovations, copying errors, drift and founder effects among dispersing groups. Selection is driven by differences in payoffs between cultural variants. Most efforts to identify these evolutionary forces in the archaeological record have relied on assessing how well the predictions from a neutral-stochastic model of cultural transmission fit a data set. Selection is inferred when the neutral-stochastic model fits poorly. A problem with this approach is that it does not test directly for the presence selection. Moreover, it does not account for the fact that both neutral-stochastic and selective forces can act at the same time on the same cultural variants. A different approach based on the Price Equation allows for the simultaneous measurement of selective and stochastic forces. This paper extends use of the Price Equation to the analysis of selective and stochastic forces operating on multiple artifact types within an assemblage. Ceramic data presented by Steele et al. (2010, Vol. 37(6): 1348–1358) from the Late Bronze Age Hittite site of Boğazköy-Hattusa, Turkey, provide an opportunity to evaluate the efficacy of this model. The results suggest that selection is a dominant process driving the frequency evolution of different bowl rim types within the assemblage and that stochastic forces played little or no role. It is also clear, however, that we should be attentive to combinations of direct and indirect selective effects within assemblages consisting of multiple artifact types.

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### 1. Introduction

Evolution is fundamentally about patterns of change in natural systems. On the surface the study of evolution sounds easy enough, but in practice it is a very difficult task. Numerous complications arise out of sampling constraints, the temporal and spatial scales of observation and the simple fact that there are many potential forces driving evolution. There are thus good historical reasons why processes such as selection, mutation, migration and drift have often been modeled independently. Selection is essentially change driven by differential payoffs or fitnesses within populations, while mutation, drift and potentially migration describe stochastic sources of change unrelated to payoffs or fitness. While conceptually different, it is clear in reality that both selective and stochastic processes are likely to be operating simultaneously.

Because systems evolve through time, it also can be difficult to detect and diagnose the operation of selection and stochastic forces in static (i.e., non-temporal or non-longitudinal) distributions of variation or diversity (Hubbell, 2001; Lande and Arnold, 1983; Templeton, 2002; Templeton et al., 1995). As recognized by Steele et al. (2010: 1350), the problems of detecting selection and stochastic forces in static samples are particularly acute if the systems in question are not at equilibrium.

Our focus in this paper is on the relationship between the rates of change and the variance in archaeological attributes, key elements of Fisher's Fundamental Theorem of Natural Selection (Frank and Slatkin, 1992; Rice, 2004). We generalize a method based the Price Equation (Brantingham, 2007), which partitions evolutionary process into payoff-correlated (selection) and payoff-uncorrelated (stochastic) components (Frank, 1997; Lande and Arnold, 1983; Price, 1970), to evaluate change through time in the frequencies of multiple related artifact types. We analyze the Hittite ceramic assemblage from Boğazköy-Hattusa, Turkey, recently published by Steele et al. (2010: 1350). We focus on the materials from this site not because of any fundamental disagreement with

\* Corresponding author. Tel.: +1 310 267 4251; fax: +1 310 206 7833.

E-mail addresses: [branting@ucla.edu](mailto:branting@ucla.edu) (P.J. Brantingham), [cperreault@ucla.edu](mailto:cperreault@ucla.edu) (C. Perreault).

<sup>1</sup> Tel.: +1 310 825 2055; fax: +1 310 206 7833.

the authors about their conclusions, but because the assemblage provides a nice focal point to discuss approaches to detecting selection and drift in archaeological context. Using the Price Equation we provide evidence confirming that selection greatly outweighs stochastic forces in driving change in the Boğazköy-Hattusa assemblage. However, we also show that bowl rim types may fall into different co-evolutionary groups and that the processes operating within these groups may not be so simply diagnosed.

This paper proceeds as follows. First, we introduce the Price Equation and its derivation. We then show how payoff-correlated and payoff-uncorrelated effects can be estimated from longitudinal archaeological data. Second, we extend the Price Equation to consider the problem of frequency evolution across multiple related types. Third, we apply the method to the ceramic assemblage from Boğazköy-Hattusa. Finally, the implications of the Price Equation for detecting patterns and processes of archaeological change within archaeological assemblages are discussed.

## 2. The Price Equation

The Price Equation provides a simple, yet powerful generalization of the forces contributing to evolutionary change in any attribute. It is a theorem which, in its general form, is true of any system of differential transmission (Frank, 1997). Here we will quickly derive the Price Equation in terms relevant to the evaluation of the Boğazköy-Hattusa ceramic assemblage. Specifically, we use the Price Equation to model change in the frequency of ceramic bowl rim types among different ceramic wares. A more exhaustive derivation in comparable archaeological terms is presented in Brantingham (2007, see also Frank, 1997; Rice, 2004). Taphonomic biases are discussed at length later in this paper.

Consider a ceramic assemblage consisting of multiple ware types each indexed by  $i$ . A ceramic ware is defined as a class of vessels that share a common technology, fabric and surface treatment. Within each ware we identify bowls with two possible rim forms, inverted and everted rims. The relative frequency of inverted rims in ceramic ware  $i$  is  $z_i = n_i/N_i$ , where  $n_i$  is the number of specimens of ware type  $i$  with inverted rims and  $N_i$  is the total number of specimens of ware  $i$ . The relative frequency of specimens with everted rims within ware  $i$  is therefore  $y_i = 1 - z_i$ . It is possible to calculate the mean frequency of bowls with inverted rims across all ware types as  $z = \sum_i p_i z_i$ , where  $p_i = N_i/N$  is the proportion of all bowls, regardless of rim type, attributed to each ware  $i$ , and  $N$  is the total count of all specimens, regardless of ware.

Now consider two different sources of change in the frequency of ceramic bowls with inverted rims. The first source of change is dependent on payoffs associated with the use of bowls with inverted rims at different frequencies within each ware. Perhaps inverted rim bowls are used by prestigious individuals, or by the majority of the group, making wares that incorporate this design more likely to be imitated and therefore reproduced by pottery makers (see Boyd and Richerson, 1985, Richerson and Boyd, 2005 for a discussion of these cultural transmission biases). Alternatively, we might assume that such bowls perform better in some mundane economical function and therefore the associated wares are more likely to be reproduced. Let  $w_i$  be the absolute payoff associated with the use of inverted rim bowls at a given frequency and, for the sake of simplicity, assume that it is a linear function of  $z_i$

$$w_i = uz_i + w_0 \quad (1)$$

Here  $u$  is a positive constant reflecting the hypothesized benefits that arise from using inverted rim bowls and  $w_0$  is some baseline

payoff. Across all ware types it is possible to calculate the mean payoff associated with the use of inverted rim bowls as  $w = \sum_i p_i w_i$ . In a set time interval of ceramic production, the payoff-correlated change in the frequency of bowls with inverted rims is

$$p'_i z_i = p_i \frac{w_i}{w} z_i \quad (2)$$

where  $w_i/w$  is the relative payoffs arising from use of inverted rim bowls, and  $p'_i$  indicates the proportion of ware type  $i$  in the next time interval (see Brantingham, 2007). In general, Equation (2) shows that when the absolute payoff  $w_i$  to ware  $i$  for using inverted rim bowls at a given frequency is greater than the mean payoff across all wares (i.e.,  $w_i/w > 1$ ), then the fraction of the ceramic assemblage made up ware  $i$  will increase. The result will be an increase in the frequency of inverted rim bowls within the assemblage  $p_i \rightarrow p'_i$ , even if the frequency of inverted rim bowls within ware  $i$  does not change  $z_i \rightarrow z_i$ . This payoff-correlated process of change is conceptually (and mathematically) equivalent to selection and, in this particular case, selection is acting at the scale of the ceramic ware.

A second source of change in the frequency of ceramic bowls with inverted rims may be uncorrelated with payoff differences. Such stochastic fluctuations in rim type frequencies may arise from the whims of potters (i.e., “style”), or copying errors (Eerkens and Lipo, 2005). In this case, we may write

$$p_i z'_i = p_i (z_i + \delta_i) \quad (3)$$

where  $\delta_i$  is some small, random change in the frequency of inverted rim bowls and  $z'_i$  indicates their frequency within ware  $i$  in the next time interval. Typically,  $\delta_i$  will be some continuous random variable drawn from a probability distribution with characteristic mean  $\mu$ , variance  $\sigma$ , and shape  $\gamma$ . Here we assume that  $\delta_i$  is drawn from a symmetrical normal distribution with characteristic mean and variance. Under such conditions, the frequency of inverted rim bowls will perform a random walk through time, a point illustrated by Neiman (1995), Brantingham (2007) and Steele et al. (2010). Note, however, that the random walk is driven entirely by stochastic fluctuations within ceramic wares  $z_i \rightarrow z'_i$ . The proportion of the assemblage made up ware  $i$  does not change  $p_i \rightarrow p_i$ . This payoff-uncorrelated process is conceptually (and mathematically) equivalent to a process of stochastic mutation or drift and, in this particular case, stochastic fluctuations occur entirely at the scale of the bowl rim type.

Equations (2) and (3) may be combined into a single specification that simultaneously accounts for both selective and stochastic forces operating over time

$$p'_i z'_i = p_i \frac{w_i}{w} (z_i + \delta_i) \quad (4)$$

Change occurs both in the proportion of the assemblage made up of ware  $i$  (i.e.,  $p_i \rightarrow p'_i$ ), and in the frequency of bowls with inverted rims within ware  $i$  ( $z_i \rightarrow z'_i$ ). The mean frequency of ceramic bowls across all wares after selection and stochastic fluctuations is then given as  $z' = \sum_i p'_i z'_i$ .

Using the above components, it is straightforward to derive the Price Equation, a simple yet powerful statement about the contributions to evolutionary change. The Price Equation partitions the rate of change in the mean value of an attribute—the mean frequency of inverted rim bowls, in our case—into components that are alternatively correlated and uncorrelated with payoffs or fitness (Brantingham, 2007; Frank, 1997; Price, 1970; Rice, 2004). It is worth restating the derivation of the Price Equation for completeness

$$\begin{aligned}
 \Delta z &= z' - z \\
 &= \sum_i p_i \frac{w_i}{w} (z_i + \delta_i) - \sum_i p_i z_i \\
 &= \sum_i p_i \frac{w_i}{w} z_i - \sum_i p_i z_i + \sum_i p_i \frac{w_i}{w} \delta_i \\
 &= \sum_i p_i \frac{w_i}{w} z_i - \sum_i p_i \frac{w_i}{w} \sum_i p_i z_i + \sum_i p_i \frac{w_i}{w} \delta_i
 \end{aligned}
 \tag{5}$$

where  $\Delta z$  is the rate of change in the mean frequency of inverted rim bowls and is calculated as the difference in the means between a younger  $z'$  and an older  $z$  archaeological assemblage. On the last line of Equation (5), note that  $\sum_i p_i w_i/w$  is equal to unity and therefore may be introduced freely. What Price (1970) then recognized was that the first and second terms of Equation (5) are equivalent to the standard covariance formula in statistics and the third term is equivalent to the expectation of the product of two random variables, namely

$$\begin{aligned}
 \Delta z &= E\left[\frac{w_i}{w}, z_i\right] - E\left[\frac{w_i}{w}\right] E[z_i] + E\left[\frac{w_i}{w}, \delta_i\right] \\
 &= \text{COV}\left[\frac{w_i}{w}, z_i\right] + E\left[\frac{w_i}{w}, \delta_i\right]
 \end{aligned}
 \tag{6}$$

Under conditions where stochastic fluctuations in the frequency of inverted rim bowls are uncorrelated with payoffs, Brantingham (2007) showed that Equation (6) reduces to

$$\Delta z = \underbrace{\text{COV}\left[w_i/w, z_i\right]}_{\text{selection}} + \underbrace{E[\delta_i]}_{\text{stochastic forces}}
 \tag{7}$$

The implication of Equation (7) is that payoff-correlated, selective forces and payoff-uncorrelated, stochastic forces may contribute simultaneously to directional change in the frequency of an archaeological attribute. It is not a foregone conclusion, however, that selection and stochastic forces *must* contribute simultaneously or in equal proportions to observed change.

### 3. Empirical evaluation of the Price Equation

An equivalent specification of the Price Equation leads to one method for estimating the strength of selective and stochastic forces in archaeological contexts (Brantingham, 2007, see also Endler, 1986; Lande and Arnold, 1983). Specifically, the covariance can be expressed as the product of the regression coefficient  $\beta_1$  of relative payoffs  $w_i/w$  on the frequency of inverted rim bowls  $z_i$ , and the variance  $\text{VAR}[z_i]$  in the frequency of inverted rim bowls

$$\Delta z = \beta_1 \text{VAR}[z_i] + E[\delta_i]
 \tag{8}$$

The variable  $\beta_1$  describes the strength of selection or selection gradient (Hoekstra et al., 2001, Kingsolver et al., 2001, Lande and Arnold, 1983), while  $E[\delta_i]$  is the expected impact of stochastic fluctuations unrelated to payoffs. If the payoff function linked to inverted rim bowls is positive, then  $\beta_1$  will also be positive—with a value of  $\sim u/w$  in the case of the payoff function in Equation (2). If stochastic fluctuations in the frequency of inverted rim bowls arises from a probability distribution with mean  $\mu$ , then  $E[\delta_i] \sim \mu$  (Brantingham, 2007).

Note that Equation (8) is in slope-intercept form. Thus, provided that one can measure archaeologically the change in mean frequency of inverted rim bowls across multiple dated stratigraphic units or assemblages, and also measure the variance in frequencies of inverted rim bowls among different ware types, then the strength of selection and stochastic forces can be estimated directly from archaeological data using simple linear least-squares regression procedures (Endler, 1986; Lande and Arnold,

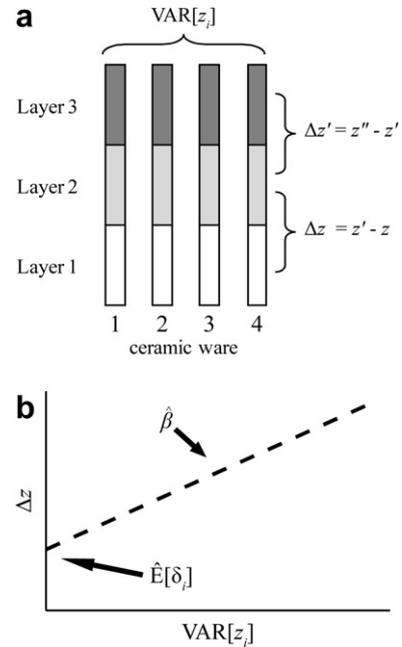
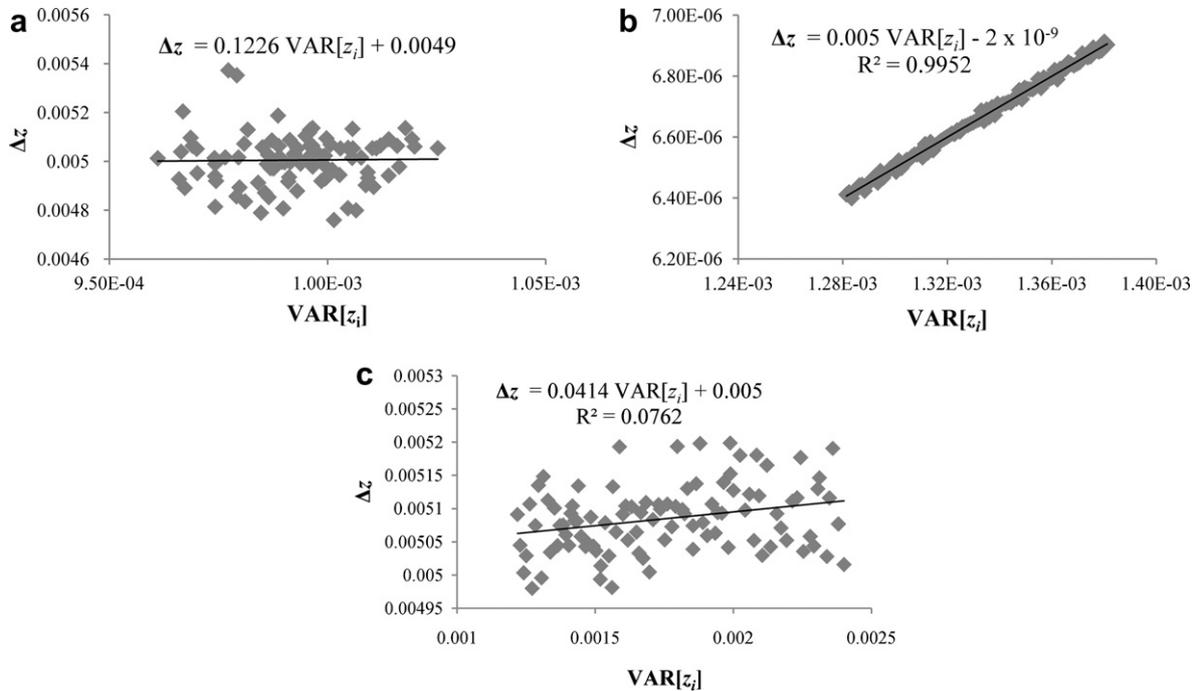


Fig. 1. Conceptual illustration of (a) how the change in mean frequency and variance in frequency of a single rim type are calculated and (b) how linear regression of these quantities leads to estimates of selection and stochastic forces.

1983). Fig. 1a illustrates conceptually how these observables are calculated for an ideal case where stratigraphic layers represent equal intervals of time. Fig. 1b illustrates how the estimation procedure is implemented. Fig. 2 demonstrates the effectiveness of the method using simulated archaeological data (see also Brantingham, 2007).

Selective and stochastic forces can be estimated with considerable accuracy (Table 1), provided that certain assumptions are met. First, if one is working with relative frequencies of attributes such as rim forms, it is important those frequencies not be too close to the extremes. The concern is that a ceramic ware that is dominant within an assemblage (i.e.,  $p_i \sim 1$ ) is not free to change under selective forces, while a ware that exhibits near complete dominance by inverted rim bowls (i.e.,  $z_i \sim 1$ ) is not free to change under stochastic forces. Selection cannot increase the proportion of the assemblage made up of this ware, while stochastic forces are constrained only to reduce the frequency of inverted rim bowls. Second, it is also the case that extreme selective or stochastic forces overwhelm the capacity of least squares regression to detect a relationship between rates of change and variance in trait frequencies. Stringent selection produces massive, discontinuous changes in attribute means from small differences in variance. Stochastic processes with high variance likewise may produce large jumps in frequency that may be confused with selection, even if the mean effect of the stochastic process is zero. Linear regression is poorly suited for such data patterns. Therefore, the simultaneous operation of selection and stochastic forces is most detectable when selection is present, but is not overwhelming, and when stochastic variations are small relative to selection. Finally, it is important to emphasize that the strength of selection  $\beta_1$  is the regression coefficient of relative payoffs  $w_i/w$  on trait frequency  $z_i$ . Since the mean payoff  $w$  is not a constant, the strength of selection is also strictly not a constant. Rather,  $\beta_1$  will tend to decline as mean payoffs increase over the course of evolution. If mean payoffs are not known independently, then corrections to  $\beta_1$  may be necessary to arrive at a quantitatively accurate estimate of the strength of selection (see Endler, 1986; Lande and Arnold, 1983).



**Fig. 2.** Illustration of the Price Equation method for estimating the strength of selection and stochastic forces operating on the evolution of the frequency of an archaeological attribute. Shown in each case is the relationship between the rate of change in the mean frequency of the trait ( $\Delta z$ ) and the variance in the trait ( $\text{VAR}[z_i]$ ) across 100 hypothetical ware types through 100 stratigraphic levels (i.e., time intervals). (a) Stochastic evolution of attribute frequency where no selective forces are at play (i.e.,  $\beta_1 = 0$ ) and stochastic fluctuations are drawn from a normal distribution with mean  $\mu = 0.005$  and standard deviation  $\sigma = 0.001$  (i.e.,  $E[\delta_i] = 0.005$ ). (b) Selection for higher frequencies of inverted rim bowls ( $\beta_1 \sim 0.005$ ) where stochastic forces play no role (i.e.,  $E[\delta_i] = 0$ ). (c) Evolution of attribute frequency where selective and stochastic forces are in operation. Here  $\beta_1 \sim 0.05$  and  $E[\delta_i] = 0.005$ . In all simulations, the total quantity of ceramic vessels across all ware types is held constant in each stratigraphic level and the focal attribute of inverted rim bowls is present in only five of the initial ware types in the lowest stratigraphic levels at random initial frequencies  $0.1 \leq z_i \leq 0.2$ .

**Fig. 2** provides a test of the ability to estimate selective and stochastic forces operating on frequency evolution when these processes are operating in isolation and simultaneously. In all cases, data represent the variance in frequency of a single rim type, for a given focal stratum, paired with the rate of change in the mean frequency between the focal stratum and the next higher stratum. One hundred stratigraphic intervals are plotted in each panel. **Fig. 2a** shows the outcome of regressing  $\Delta z$  on  $\text{VAR}[z_i]$  when stochastic fluctuations are the only process responsible for changes in rim frequencies. In this case, the true model parameters are  $\beta_1 = 0$  and  $E[\delta_i] = \mu = 0.005$ . Linear least squares regression estimates these values as  $\hat{\beta}_1 = 0.1226$  and  $\hat{E}[\delta_i] = 0.0049$ . Note, however, that the estimated value of  $\hat{\beta}_1$  is not significantly different from zero ( $t = 0.1634$ ,  $p\text{-value} = 0.8705$ ), suggesting correctly that selection is not at play in this evolutionary sequence. The estimated intercept value is statistically significant ( $t = 6.5568$ ,  $p\text{-value} \ll 0.0001$ ) and is very close to the true value. Stochastic drift can be accurately identified and estimated using this technique.

**Fig. 2b** shows an alternative example where stochastic fluctuations are not present, but selection is operating. The underlying payoff function is  $w_i = uz_i + w_0$  with  $u = 0.005$  and  $w_0 = 1$ . The true parameter values for the model are  $\beta_1 \sim 0.005$  and  $E[\delta_i] = 0$ . Linear least squares regression estimates these values as  $\hat{\beta}_1 = 0.005$  and  $\hat{E}[\delta_i] = -1.74 \times 10^{-9}$ . Only the estimate for  $\hat{\beta}_1$  is significantly different from zero ( $t = 141.74$ ,  $p\text{-value} \ll 0.0001$ ). Selection can also be accurately estimated using this technique.

**Fig. 2c** presents a case where selection and stochastic forces are operating simultaneously. Here the true values for the model parameters are  $\beta_1 \sim 0.05$  and  $E[\delta_i] = 0.005$ . These are estimated in the linear regression as  $\hat{\beta}_1 = 0.0414$  ( $t = 2.8138$ ,  $p\text{-value} = 0.0059$ ) and  $\hat{E}[\delta_i] = 0.005$  ( $t = 190.2455$ ,  $p\text{-value} \ll 0.00001$ ). It is therefore feasible to identify and estimate the simultaneous operation of selection and stochastic processes operating within archaeological assemblages. Note, however, that the coefficient of determination ( $R^2 = 0.0762$ ) is very small, indicating that only limited proportion of the variance in rates of change is explained by variance of the frequency of inverted rim bowls, which is attributed to selection.

**Table 1**

Regression statistics for Price Equation estimates of stochastic and selective forces in three simulated control cases.

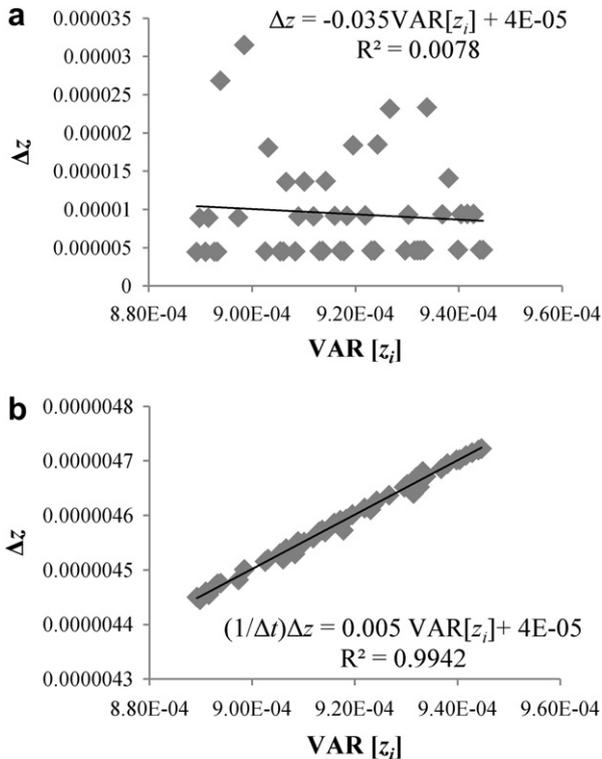
	Simulated Value	Estimates	Standard Error	$t$	$p$ -value	Lower 95%	Upper 95%
Stochastic							
$E[\delta_i]$ (intercept)	0.005	0.0049	0.0007	6.5568	2.76E-09	0.0034	0.0064
$\beta_1$ (slope)	0	0.1226	0.7501	0.1634	0.8705	-1.3663	1.6115
Selective							
$E[\delta_i]$	0	-1.74E-09	4.70E-08	-0.0371	0.9705	-9.50E-08	9.15E-08
$\beta_1$	0.005	0.0050	3.53E-05	141.7417	2.61E-113	0.0049	0.0051
Stochastic & Selective							
$E[\delta_i]$	0.005	0.0050	2.63E-05	190.2455	1.55E-125	0.0050	0.0051
$\beta_1$	0.05	0.0414	0.0147	2.8138	0.0059	0.0122	0.0707

The fraction of unexplained variance ( $1 - R^2 = 0.9238$ ) must be attributable to stochastic forces since we know that this is the only other source of change within the controlled simulation. Interpretation of  $R^2$  in real archaeological contexts will certainly be more complicated.

While conceptually useful, the above assumption that stratigraphic layers represent equal intervals of time is clearly impractical for most archaeological settings. However, where precise geochronological information is available to determine the absolute ages of stratigraphic layers, and therefore the absolute time intervals represented between archaeological assemblages, it is possible to modify the Price Equation to accommodate temporally non-uniform sequences. Let  $\Delta t = t' - t$  be the absolute amount of time separating an overlying stratigraphic unit dated to  $t'$  and an underlying stratigraphic unit dated to  $t$ . The inverse  $\tau/\Delta t$  is a rate correction where  $\tau$  is a constant corresponding to the time scale of interest. The relevant equation for estimating the strength of stochastic and selective forces may be rewritten as

$$\frac{\tau}{\Delta t} \Delta z = \beta_1 \text{VAR}[z_i] + E[\delta_i] \quad (9)$$

In general, stratigraphic layers separated by more time are expected to display a greater observed rate of change (i.e.,  $\Delta z$  is large when  $\Delta t$  is large). Weighting by the inverse of the absolute time between layers standardizes observed rates of change to a uniform time scale  $\tau$ . Fig. 3 demonstrates the necessity of such a correction in the case where the time intervals between



**Fig. 3.** A correction term must be introduced into the Price Equation to account for differences in the absolute amount of time represented between stratigraphic intervals. Both (a) and (b) show data from a simulation where underlying selective process favors higher frequencies of inverted rim bowls ( $\beta_1 \sim 0.005$ ) and stochastic forces play no role (i.e.,  $E[\delta_i] = 0$ ). The time between stratigraphic intervals is variable, ranging from one to seven temporal units (i.e.,  $\Delta t_{\min} = 1$  and  $\Delta t_{\max} = 7$ ). In (a) it is incorrectly assumed that the time between each stratigraphic interval is uniform at one temporal unit. The resulting regression leads to erroneous results. In (b) a correction  $1/\Delta t$  is applied to each observed value of  $\Delta z$ , which yields a regression with the correct parameter estimates (see Fig. 2b).

stratigraphic assemblages is non-uniform. Using simulated data in which selection is the only force responsible for change in rim frequencies (i.e.,  $\beta_1 \sim 0.005$ ,  $E[\delta_i] = 0$ ) (see Fig. 2b), we chose a random sample of 49 stratigraphic intervals from which to calculate values of  $\Delta z$  and  $\text{VAR}[z_i]$ . The natural time scale of change in this simulation is a single stratigraphic interval, or  $\tau = 1$ , and an unbiased stratigraphic sequence should have  $\Delta t = 1$  for each and every pair of strata from which  $\Delta z$  is calculated. Our random sample of stratigraphic units yields observed values of  $\Delta t$  ranging from  $\Delta t_{\min} = 1$  to  $\Delta t_{\max} = 7$ , and correction terms ranging from  $1/\Delta t_{\min} = 1$  to  $1/\Delta t_{\max} = 0.143$ . Attempting to fit the Price Equation to the random sample of stratigraphic *without* a corresponding temporal correction yields erroneous results (Fig. 3a). Multiplying each observed value of  $\Delta z$  by its corresponding correction  $1/\Delta t$  followed by linear regression recovers the correct parameter values (Fig. 3b).

#### 4. Frequency evolution among multiple artifact types

The method described above is appropriate for estimating the strength of selective and stochastic forces operating on a single archaeological attribute. It is reasonable to ask whether the method is also applicable to the study of frequency changes across multiple related attributes. Here we develop conceptually how the Price Equation may be deployed in analysis of frequency changes across  $k$  artifact types (see also Endler, 1986; Lande and Arnold, 1983). The intention is to develop a method appropriate to the analysis of the nine different bowl rim forms recognized at the Boğazköy-Hattusa site (Steele et al., 2010: 1350).

First, using our hypothetical example developed above, consider the Price Equation from the point of view of the frequency of everted rim bowls. Since this quantity is the complement of the frequency of inverted rim bowls (i.e.,  $y_i = 1 - z_i$ ), it is necessarily the case that any selective and stochastic forces that increase the mean frequency  $z$  of inverted rim bowls must be offset exactly by a decrease in the mean frequency of everted rim bowls. Indeed, by substituting  $z_i = 1 - y_i$  into Equation (5) and changing the sign of  $\delta_i$  to reflect the fact that a stochastic fluctuation in one direction for  $z_i$  is automatically a fluctuation in the opposite direction for  $y_i$ , it can be shown that

$$\Delta y = \text{COV}\left[\frac{w_i}{w}, y_i\right] - E[\delta_i] \quad (10)$$

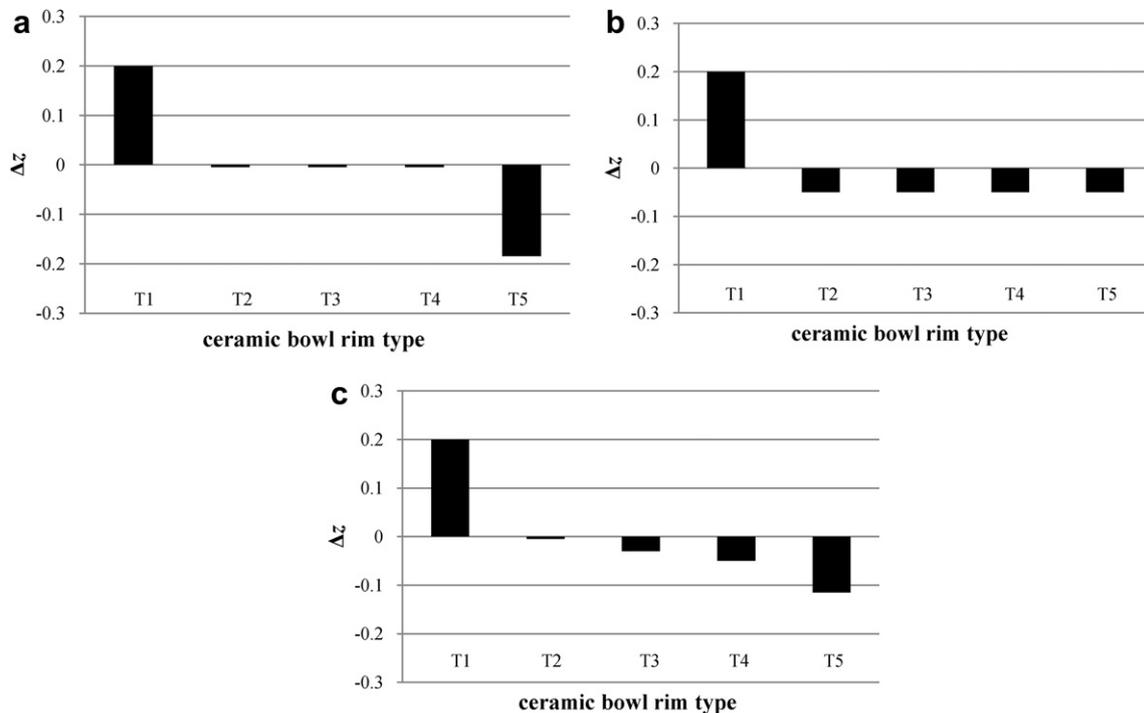
Recalling that  $w_i/w$  is relative payoff associated with the frequency of inverted rim bowls  $z_i$ , the first term of Equation (10) will be negative if it is positive in the case of Equation (7). In other words,  $\text{COV}[w_i/w, y_i] = -\text{COV}[w_i/w, z_i]$ .

More generally, for ceramic bowls with  $k$  different rim types it can be shown that

$$\Delta z^{(k)} = - \sum_{j \neq k} \Delta z^{(j)} \quad (11)$$

Equation (11) says that any change in the mean frequency  $z^{(k)}$  of a rim type  $k$  must be balanced by complementary changes in the frequencies of all other rim types  $j \neq k$ . The superscripts are enclosed in parentheses to emphasize that they are indices for different rim types, not exponents. For example, in an assemblage of bowls exhibiting five different rim types, if inverted rims increase in frequency by 20%, then the cumulative change across the remaining four rim types must amount to a decrease of  $-20\%$ . What Equation (11) does not say, however, is how change across the  $j$  other rim types should be distributed.

Fig. 4 illustrates several hypothetical patterns of balanced change in rim type frequencies within an assemblage. For example,



**Fig. 4.** Hypothetical distributions of change in mean frequencies across five bowl rim types. The first rim type (T1) shows an increase in mean frequency of 10%. In each hypothetical case (a, b, c), the change across T2, T3, T4 and T5 rim types amounts to a cumulative decrease of 10%. There are many possible ways in which this constraint can be satisfied, with potentially different evolutionary implications.

it is possible that a substantial change in the frequency of one rim type may be balanced predominantly by change in only one other rim type, with little or no change in other types within the assemblage (Fig. 4a). Substantial change in one type could be balanced by approximately equal levels of change among other types (Fig. 4b). Alternatively, substantial change in the frequency of one type could be matched by smooth, but declining changes among others (Fig. 4c). The number of qualitatively different patterns of change across assemblages is not limitless, but is nonetheless probably very large. The evolutionary implications of different distributions of frequency changes will be discussed at the close of this paper.

## 5. Selective and stochastic processes at Boğazköy-Hattusa

We now turn our attention to measuring selective and stochastic sources of change in a real archaeological context. We analyze the data presented by Steele et al. (2010) on rim type frequencies among ceramic bowls from the Hittite site of Boğazköy-Hattusa, Turkey. Located on the central Anatolian Plateau, Boğazköy-Hattusa is a large (maximally 180 ha) urban settlement with archaeological deposits spanning the Early Chalcolithic to the Iron Age (Glatz, 2009; Mielke et al., 2006; Schoop, 2006; Seeher, 2006). The most significant occupations date to the Late Bronze Age (ca. 2000–1200 BCE), during which time Boğazköy-Hattusa served as a capital of the Hittite Empire and developed an intricate internal organization consisting of several temple complexes, reservoirs, residential districts and ceramic production areas, all surrounded by a fortification wall (see Mielke et al., 2006).

Steele et al. (2010) concentrate on two ceramic assemblages from the Late Bronze Age Upper City, the older Oberstadt 3 and younger Oberstadt 2 phases. These assemblages derive primarily from the central Temple Quarter (Steele et al., 2010), but may include a diversity of local archaeological contexts (see Seeher, 2006). Contradictions have emerged in the dating of the Upper

City at Boğazköy-Hattusa. Some researchers favor the traditional view that the entire Upper City sequence dates to the last decades of the 13th Century BCE, while others now favor the view, derived from careful stratigraphic, radiocarbon and ceramic seriation analyses, that the Upper City developed over several centuries beginning in the late 16th Century or early 15th Century BCE. The Upper City at Boğazköy-Hattusa was in decline by the end of the 13th Century BCE (Seeher, 2006). Acknowledging this controversy, we assume that the ceramic assemblages from Oberstadt 3 and 2 phases at Boğazköy-Hattusa differ in age by 50 years.

Steele et al. (2010) focus their analysis on nine primary rim types observed across four ware types. Their purpose was to evaluate whether the distributions of rim type frequencies are statistically consistent with neutral drift models. Steele et al. (2010) rely on measures introduced by Neiman (1995), Ewens (1972) and Slatkin (1994, 1996) developed in the context of the neutral theory of genetic evolution (see Kimura, 1983). They find some evidence that rim types do not follow frequency distributions consistent with neutral expectations and therefore suggest that selection is driving their evolution.

The rim types analyzed by Steele et al. (2010) are aggregates based on a much finer typological scheme consisting of 61 variants. The nine aggregate types include: Type I1, bowls with simple rounded rims; Type I2, bowls with simple thickened rims; Type I3, bowls with inverted rims; Type I4, bowls with everted rims; Type I6, bowls with everted rims; Type I7, bowls with everted rims; Type I8, carinated bowls with everted rims; and Type I9, bowls with inverting walls. No Type I7 rim sherds were identified in the Oberstadt 3 phase making it impossible to calculate a meaningful rate of change in frequencies of this rim type (see below). We therefore drop Type I7 from further consideration, leaving eight primary rim types. The four ceramic ware types analyzed by Steele et al. (2010) include: Ware A, red/brown, medium to coarse fabric, plane ware; Ware C, fine fabric, red slipped ware; Ware D, fine to coarse fabric, beige to white slipped ware; and Ware E, fine, beige to

**Table 2**

Counts of rim sherd types by ceramic ware from the Oberstadt 3 and 2 phases at Boğazköy-Hattusa (after Steele et al., 2010: Table 1).

	Ware A (plain coarse)	Ware C (red slip)	Ware D (white slip)	Ware E (plain finer)	Total
O.St.3					
Type 11 (bowls with simple rounded rims)	80	111	28	141	360
Type 12 (bowls with simple Thickened rims)	22	12	8	36	78
Type 13 (bowls with inverted rims)	171	276	53	214	714
Type 14 (bowls with everted rims)	22	19	5	19	65
Type 15 (bowls with everted rims)	40	16	5	37	98
Type 16 (bowls with everted rims)	7	2	1	6	16
Type 18 (carinated bowls with everted rims)	5	11	2	17	35
Type 19 (bowls with inverting walls)	6	7	2	11	26
Total	353	454	104	481	1392
O.St.2					
Type 11 (bowls with simple rounded rims)	590	32	3	61	686
Type 12 (bowls with simple thickened rims)	94	5	0	52	151
Type 13 (bowls with inverted rims)	240	35	4	68	347
Type 14 (bowls with everted rims)	300	6	1	15	324
Type 15 (bowls with everted rims)	501	6	1	11	519
Type 16 (bowls with everted rims)	5	0	0	3	8
Type 18 (carinated bowls with everted rims)	4	0	0	7	11
Type 19 (bowls with inverting walls)	11	0	0	0	11
Total	1745	86	9	217	2057

red fabric, smooth or plain surfaces. Steele et al.'s (2010) summary of sherd counts by rim type and ceramic ware are shown in Table 2 and relative frequencies of each type  $z_i^{(k)}$  and proportional representation of ware  $p_i$  are shown in Table 3.

Taphonomic and sampling biases are of considerable concern attempting to document evolutionary patterns and processes from archaeological and paleontological data (Brantingham, 2007; Brantingham et al., 2007; Hunt, 2004; Lyman, 2003; Perreault, in press; Surovell and Brantingham, 2007). It is not feasible at this time to evaluate the impact of post depositional mixing, depositional time averaging, loss through erosion or sampling errors at Boğazköy-Hattusa. Steele et al. (2010) were careful to assess the impact of ceramic fragmentation on the Boğazköy-Hattusa rims sherd assemblage. They found no systematic relationship between bowl diameter and number of rim fragments, contrary to expectations that larger bowls on average will generate more ceramic sherds. This result is reassuring, but may not completely resolve the issue. We can show, however, that differential fragmentation rates among ceramic wares will have a negligible

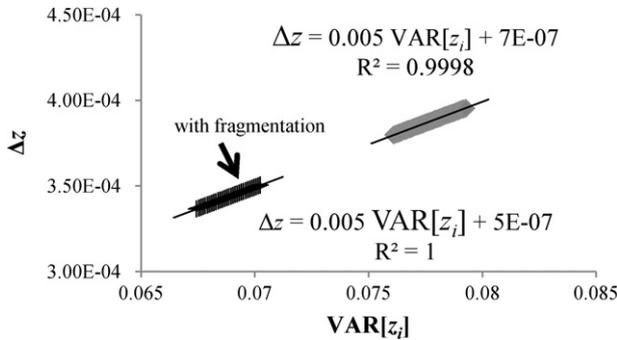
impact on the methodology presented here. If we assume that each ceramic ware  $i$  has a fragmentation rate  $\varphi_i \geq 1$ , meaning that each whole vessel produces one or more sherds, and that the fragmentation rate for a ware  $i$  does not change substantially across stratigraphic intervals, then the impact of fragmentation on the proportion of bowls of ware  $i$  given  $\varphi_i$  is approximately a constant

$$p_{i|\varphi} = \frac{\varphi_i N_i}{\sum_i \varphi_i N_i} \quad (12)$$

The component  $N_i/N$  (i.e.,  $p_i$  without fragmentation) may change through time as a result of selective forces, but the fragmentation rates will have only a fixed effect on empirical values of  $\Delta z^{(k)}$  and  $\text{VAR}[z_i^{(k)}]$ . Fig. 5 compares simulations of two hypothetical ceramic sequences where inverted rim bowls are favored by selection (i.e.,  $\beta_1 \sim 0.005$ ; see Fig. 2b). In one data series, calculations of  $\Delta z^{(k)}$  and  $\text{VAR}[z_i^{(k)}]$  are based on whole vessels. In the other series, each ware was subjected to a randomly drawn, but constant fragmentation

**Table 3**Rim type frequencies  $z_i^{(k)}$  and proportional representation of each ceramic ware  $p_i$  from the Oberstadt 3 and 2 phases at Boğazköy-Hattusa.

	Ware A (plain coarse)	Ware C (red slip)	Ware D (white slip)	Ware E (plain finer)
O.St.3				
Type 11 (bowls with simple rounded rims)	0.2266	0.2445	0.2692	0.2931
Type 12 (bowls with simple thickened rims)	0.0623	0.0264	0.0769	0.0748
Type 13 (bowls with inverted rims)	0.4844	0.6079	0.5096	0.4449
Type 14 (bowls with everted rims)	0.0623	0.0419	0.0481	0.0395
Type 15 (bowls with everted rims)	0.1133	0.0352	0.0481	0.0769
Type 16 (bowls with everted rims)	0.0198	0.0044	0.0096	0.0125
Type 18 (carinated bowls with everted rims)	0.0142	0.0242	0.0192	0.0353
Type 19 (bowls with inverting walls)	0.0170	0.0154	0.0192	0.0229
$p_i$	0.254	0.326	0.075	0.345
O.St.2				
Type 11 (bowls with simple rounded rims)	0.3381	0.3721	0.3333	0.2811
Type 12 (bowls with simple thickened rims)	0.0539	0.0581	0.0	0.2396
Type 13 (bowls with inverted rims)	0.1375	0.4070	0.4444	0.3134
Type 14 (bowls with everted rims)	0.1719	0.0930	0.1111	0.0691
Type 15 (bowls with everted rims)	0.2871	0.0698	0.1111	0.0507
Type 16 (bowls with everted rims)	0.0029	0.0	0.0	0.0138
Type 18 (carinated bowls with everted rims)	0.0023	0.0	0.0	0.0323
Type 19 (bowls with inverting walls)	0.0063	0.0	0.0	0.0
$p_i$	0.848	0.042	0.004	0.106



**Fig. 5.** Fragmentation of ceramic vessels has limited impact on measurement of selective and stochastic forces when fragmentation rates differ across ceramic wares, but are constant through time. Shown is the relationship between the rate of change in the mean frequency of the trait ( $\Delta z$ ) and the variance in the trait ( $\text{VAR}[z_i]$ ) across four hypothetical ware types through 40 stratigraphic levels (i.e., time intervals). Selection is for higher frequencies of inverted rim bowls ( $\beta_1 \sim 0.005$ ) where stochastic forces play no role (i.e.,  $E[\delta_i] = 0$ ) (see Fig. 2b). Rates of change and measured variance are higher for whole vessels (gray boxes) compared with fragmented vessels (black crosses). However, the estimates for the strength of selection are identical. Fragmentation rates  $\phi_i$  for each ware  $i$  were randomly drawn between  $1 \leq \phi_i \leq 25$  and held constant over the stratigraphic sequence.

rate  $1 \leq \phi_i \leq 25$ . For example, Ware A might generate exactly 23 sherds from each whole vessel, in each stratigraphic interval, while Ware C might generate only seven sherds per whole vessel. Fragmentation impacts the absolute values of  $\Delta z^{(k)}$  and  $\text{VAR}[z_i^{(k)}]$ , but the regression of  $\Delta z^{(k)}$  on  $\text{VAR}[z_i^{(k)}]$  yields identical estimates for  $\beta_1$ . Similar results would apply in a more realistic case where fragmentation rates for each unique ceramic ware may vary in each stratigraphic unit according to a stationary probability distribution with some mean  $\mu(\phi_i)$  and variance  $\sigma(\phi_i)$ . The results will simply be noisier compared with constant fragmentation rates. In general, therefore, sherd counts and whole vessel counts are expected to lead to similar results. We may talk about selection and stochastic processes operating at the scale of ceramic wares and whole vessels while analyzing rim sherd fragments.

The Price Equation requires that we calculate the mean and variance in rim type frequencies and the change in mean frequencies through time (Brantingham, 2007). The results are presented in Table 4 using the raw rim sherd frequencies from Boğazköy-Hattusa. To illustrate how these values are calculated

from the data in Table 2 and Table 3, consider simple rounded rim bowls (Type I1). There are 80 rim sherds of this type among the 353 specimens classified as Ware A in the Oberstadt 3 phase. The relative frequency of Type I1 rims among Ware A sherds is therefore  $z_A^{(1)} = 0.227$ . On average, 22.7% of bowl rim sherds in Ware A have simple rounded rims. The corresponding frequencies of Type I1 rim sherds among the other ceramic wares are  $z_C^{(1)} = 0.244$ ,  $z_D^{(1)} = 0.269$ ,  $z_E^{(1)} = 0.293$ . Of the 1392 bowls found in Oberstadt 3, a total of 353 are Ware A, yielding a proportion  $p_A = 0.254$ . On average, 25.4% of all rim sherds, regardless of rim type, are Ware A. The corresponding proportions for the other ceramic wares are  $p_C = 0.326$ ,  $p_D = 0.075$ , and  $p_E = 0.346$ . The mean frequency of Type I1 rims across all ware types is then  $z^{(1)} = \sum_i p_i z_i^{(1)} = [(0.254 \times 0.227) + (0.326 \times 0.244) + (0.075 \times 0.269) + (0.346 \times 0.293)] = 0.259$ , restating for clarity that the subscript  $i$  indexes ware type and the superscript indexes rim type. The variance in the frequency of simple rounded rim bowls in the Oberstadt 3 phase assemblage is calculated using the formula  $\text{VAR}[z_i^{(1)}] = \sum_i p_i (z_i^{(1)} - z^{(1)})^2 = [0.254(0.227 - 0.259)^2 + 0.326(0.244 - 0.259)^2 + 0.075(0.269 - 0.259)^2 + 0.346(0.293 - 0.259)^2] = 0.00074$ . The change in the mean frequency of simple rounded rim bowls between the Oberstadt 3 (older  $z$ ) and Oberstadt 2 (younger  $z'$ ) phases is then  $\Delta z^{(1)} = z' - z = 0.333 - 0.259 = 0.075$ . Analogous calculations are made for each of the other seven rim types, and values of the uncorrected  $\Delta z^{(k)}$  and time-scale corrected  $(\tau/\Delta t)\Delta z^{(k)}$  are presented (Table 4). The correction is based on an assumed 50 years separating the Oberstadt 3 and 2 assemblages. Finally, we calculate an empirical estimate of the expected value of stochastic fluctuations in rim type frequencies as  $E[\delta_i] = \sum_i p_i (z_i^{(k)'} - z_i^{(k)})$ , where  $z_i^{(k)}$  is the raw frequency of rim type  $k$  in the Oberstadt 3 assemblage and  $z_i^{(k)'}$  is the raw frequency in the Oberstadt 2 assemblage.

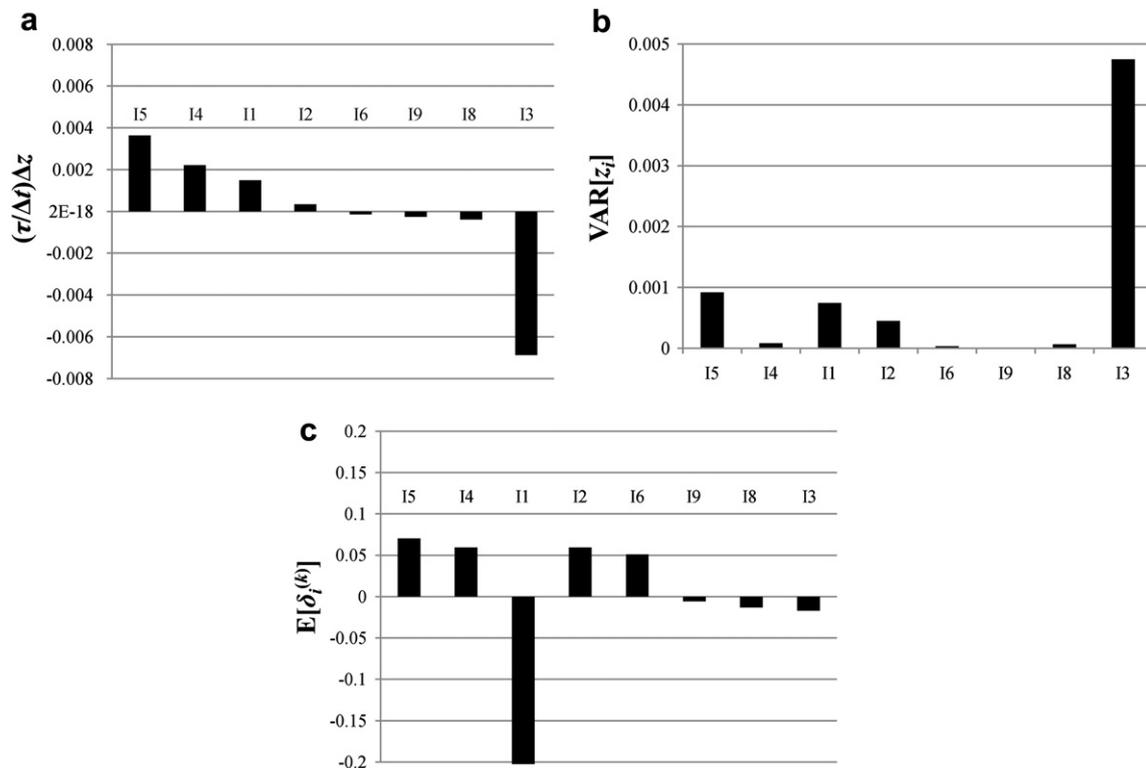
Inspection of Table 4 shows that half of the rim sherd types display increases in mean frequency and half decreases. While the total amount of positive change is balanced by the total amount of negative change, as required by Equation (11), the distribution of change is not uniform across the eight rim types (Fig. 6a). Rather, sherds with inverted rims (Type I3) show a massive decline in frequency ( $\Delta z^{(3)} = -0.344$ ), four types show small amounts of change both in positive and negative directions (Types I2, I6, I9 and I8), and three rim types show modest positive increases in frequency (Types I1, I4 and I5). Type I5 bowls with everted rims

**Table 4**  
Mean, variance, change in the mean frequency, and the expected stochastic change of bowl rim types among four ceramic ware types at Boğazköy-Hattusa.

	$z$	$\text{Var}[z_i]$	$\Delta z$	$(\tau/\Delta t)\Delta z^a$	$E[\delta_i]^b$
O.St.3					
Type I1 (bowls with simple rounded rims)	0.25862069	0.00074478	0.074874692	0.001497494	0.070518187
Type I2 (bowls with simple thickened rims)	0.056034483	0.00045069	0.017373393	0.000347468	0.059391878
Type I3 (bowls with inverted rims)	0.512931034	0.00474934	-0.344238764	-0.006884775	-0.203830519
Type I4 (bowls with everted rims)	0.046695402	8.7617E-05	0.110815536	0.002216311	0.059428621
Type I5 (bowls with everted rims)	0.070402299	0.0009221	0.181906889	0.003638138	0.050977648
Type I6 (bowls with everted rims)	0.011494253	3.4606E-05	-0.007605094	-0.000152102	-0.005990495
Type I8 (carinated bowls with everted rims)	0.025143678	6.9401E-05	-0.019796085	-0.000395922	-0.013415734
Type I9 (bowls with inverting walls)	0.018678161	1.0274E-05	-0.013330567	-0.000266611	-0.017079587
O.St.2					
Type I1 (bowls with simple rounded rims)	0.333495382	0.00036988	—	—	—
Type I2 (bowls with simple thickened rims)	0.073407876	0.00327202	—	—	—
Type I3 (bowls with inverted rims)	0.16869227	0.00573802	—	—	—
Type I4 (bowls with everted rims)	0.157510938	0.00118354	—	—	—
Type I5 (bowls with everted rims)	0.252309188	0.0067958	—	—	—
Type I6 (bowls with everted rims)	0.003889159	1.2002E-05	—	—	—
Type I8 (carinated bowls with everted rims)	0.005347594	8.5635E-05	—	—	—
Type I9 (bowls with inverting walls)	0.005347594	5.113E-06	—	—	—

<sup>a</sup> Time-scale correction  $\tau/\Delta t = 1/50$ , giving the annualized rate of change.

<sup>b</sup> Calculated empirically as  $p_i [z_i^{(k)'} - z_i^{(k)}]$ .



**Fig. 6.** (a) Distribution of values for change in the mean frequency  $\Delta z$  of bowl rim types between the Oberstadt 3 and Oberstadt 2 phases at Boğazköy-Hattusa. (b) Variance in the frequency of rim types in the Oberstadt 3 phase. (c) Average change in the raw frequency of rim types between the Oberstadt 3 and Oberstadt 2.

show the most significant positive increase in frequency, jumping by  $\Delta z^{(5)} = 0.182$ .

Selection, stochastic forces, or both processes may be driving the observed changes in mean frequencies of bowl rim types at Boğazköy-Hattusa. Fig. 6b and c show, respectively, the variance in rim type frequencies in the Oberstadt 3 assemblage and our empirical estimate of stochastic fluctuations  $E[\delta_i]$  between Oberstadt 3 and 2. Visual comparisons suggest a stronger relationship between the variance in rim type frequencies and changes in mean rim type frequencies (Fig. 6a and b). In general, the change in mean rim type frequency is high when the variance in rim type frequencies is high (Fig. 6b). The one exception is rim Type 14. By contrast, there are two instances where a change in the mean rim type frequency (Fig. 6a) are paired with stochastic shifts in raw frequencies in the *opposite* direction (e.g., Types 11 and 16), or where the direction of change is correct but the magnitude of this change is very different (e.g., Types 13 and 16) (Fig. 6c). The impression is that variance in rim type frequencies makes a larger contribution to change than stochastic fluctuations.

Application of the Price Equation to the Boğazköy-Hattusa assemblage may allow further evaluation of the above observations. Ideally one would examine the relationship between variance and change in the mean frequency of attributes over multiple temporal intervals (i.e., longitudinal analysis). At Boğazköy-Hattusa, however, the data describe the variance and change in mean frequency of multiple rim types over a single temporal interval. This situation is not unlike the single selective events analyzed by Lande and Arnold (1983). We may plot together the change in mean frequency  $\Delta z^{(k)}$  against the variance in frequency  $\text{VAR}[z_i^{(k)}]$  for each rim type  $k$  and under some circumstances arrive at accurate estimates of the strength of selection. As discussed below, however, it is unlikely that we will be able to estimate the quantitative value of stochastic fluctuations, though we may still be able to identify when stochastic forces are dominant and their likely direction of impact.

The results of plotting  $\Delta z^{(k)}$  against the variance in frequency  $\text{VAR}[z_i^{(k)}]$  for the Oberstadt 3 and 2 assemblages at Boğazköy-Hattusa are presented in Fig. 7. Several immediate observations are warranted. First, it is generally true for the Boğazköy-Hattusa assemblage that rates of change are higher for higher levels of observed variance (Fig. 7). This is the expected relationship if selection were operating on the assemblage. As demonstrated above using simulated data, it is not expected if only stochastic evolutionary forces are at play (see Fig. 2a). Second, there appears to be higher-order structure to evolutionary patterns within the assemblage. On the one hand, Type 13 sherds with inverted rims stand out as having both a high variance and a large decrease in mean frequency between the two archaeological phases. On the other, a linear relationship between variance and rate of change appears to characterize the remaining rim types. Indeed, it seems reasonable to suggest that the rim types, exclusive of Type 13, display a co-evolutionary relationship that sets them apart from the evolutionary processes operating on Type 13. We arrive at this observation by visual inspection. However, it also is supported by simulation results.

Fig. 8 presents three examples of evolutionary change within an assemblage of simulated ceramic rim types. To be consistent with Boğazköy-Hattusa, the simulated assemblages include eight rim types distributed across four ceramic wares. The same variance structure is used at the start of each simulated stratigraphic sequence (i.e., identical initial conditions) and, though randomly determined, this variance structure is qualitatively similar to that observed at Boğazköy-Hattusa (see Fig. 6b). Calculations of  $\text{VAR}[z_i^{(k)}]$  are made in stratum 75 of a simulated sequence totaling 100 stratigraphic units, while  $\Delta z^{(k)}$  is calculated between stratum 75 (older) and 76 (younger).

In Fig. 8a, only one of the simulated rim types (Type 1) is subject to negative selection  $\beta_1 \sim -0.008$ , while all other rim types are not under selection (i.e.,  $\beta_{j \neq 1} = 0.0$ ). There are no stochastic

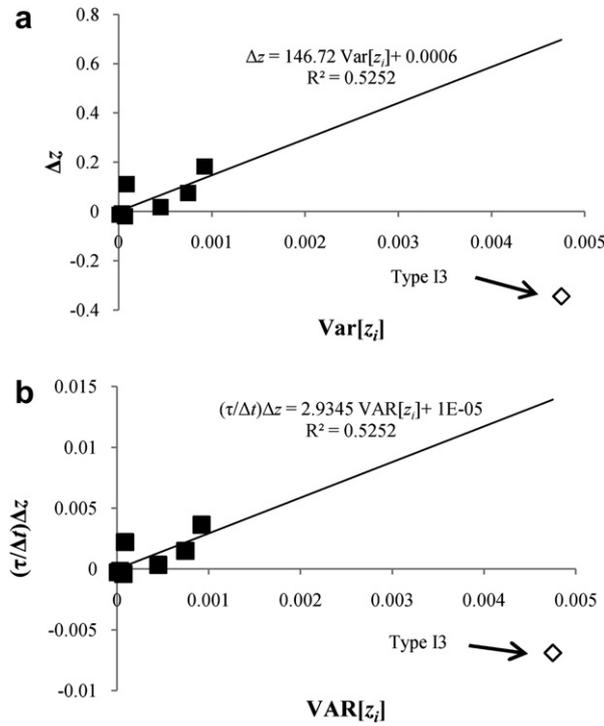


Fig. 7. Relationship between variance and the rate of change in mean frequencies for eight bowl rim types from Boğazköy-Hattusa. (a) Variance and rate of change values uncorrected for the amount of time represented between Obserstadt 3 and Obserstadt 2. (b) Variance and rate of change values corrected according to Equation (9) with  $\tau/\Delta t = 1/50$ .

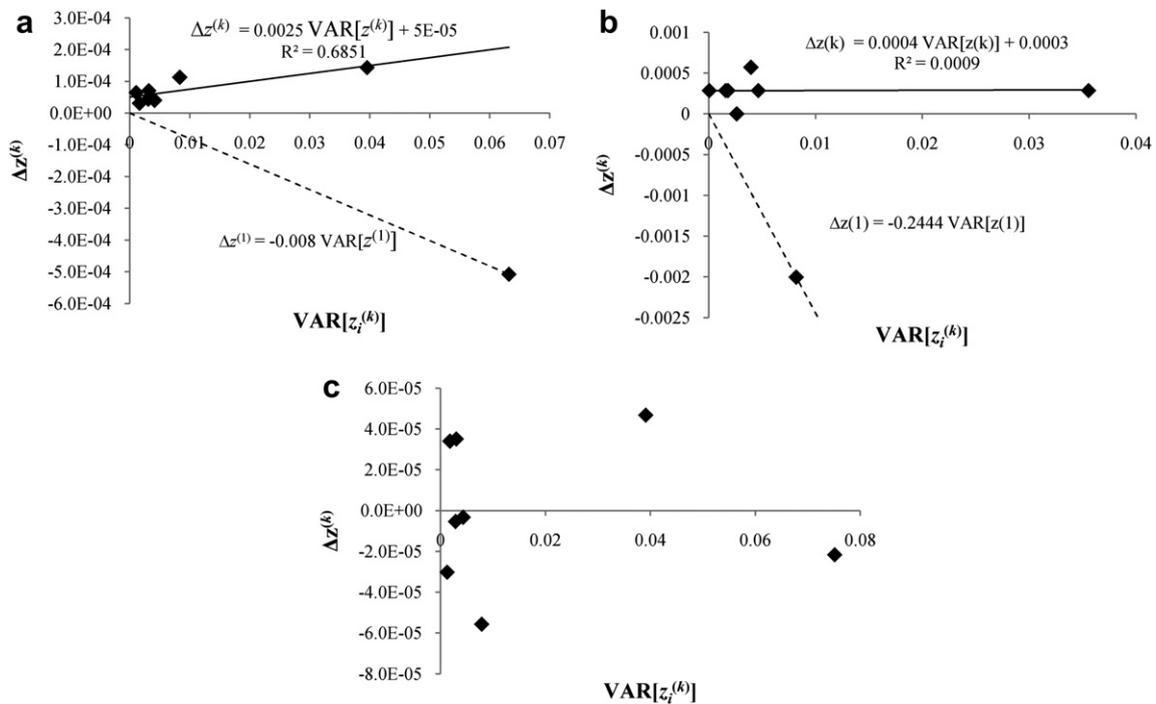


Fig. 8. Relationship between the variance and rate of change in the mean frequencies for eight simulated rim types across two stratigraphic intervals (time steps). (a) Variance and rate of change where one type experiences negative selection  $\beta_1 \sim -0.008$  and all other types are not under selection (i.e.,  $\beta_{k \neq 1} = 0.0$ ). The exact same relationship between variance and rate of change emerges if one type is not under selection  $\beta_1 = 0.0$  and all other types experience the exact same level of positive selection (i.e.,  $\beta_{k \neq 1} = 0.008$ ). (b) Only one type experiences directional stochastic fluctuations in frequency with  $\delta_i^{(1)}$  drawn from a normal distribution with mean  $\mu = -0.008$  and  $\sigma = 0.0001$  (i.e.,  $E[\delta_i^{(1)}] = -0.008$ ). (c) All types experience neutral-stochastic fluctuations in frequency with  $\delta_i^{(k)}$  drawn from a normal distribution with mean  $\mu = 0.0$  and  $\sigma = 0.0001$  (i.e.,  $E[\delta_i^{(k)}] = 0.0$ ). All simulations are based on an the same initial random variance structure qualitatively similar to that observed at Boğazköy-Hattusa. Calculations are based on observations of simulated assemblages in stratum 75 (older) and 76 (younger) from a section consisting of 100 total stratigraphic units.

fluctuations in rim frequencies (i.e.,  $E[\delta_i] = 0.0$ ). Type 1 shows a relatively large, negative rate of change, while the remaining types not under selection follow a common linear relationship with a statistically significant slope  $\hat{\beta}_{j \neq 1} = 0.0072$  ( $t = 3.2981$ ,  $p = 0.022$ ) and intercept  $\hat{E}[\delta_i] = 0.000051$  ( $t = 4.376573839$ ,  $p = 0.0072$ ). The slope connecting the solitary point for Type 1 to the origin is equal to the true selective strength as implemented in the simulation (i.e.,  $\beta_1 = -0.008$ ).

Surprisingly, the identical result is obtained when Type 1 is not under selection (i.e.,  $\beta_1 = 0.0$ ) and all of the remaining types are subject to the same, complimentary selective pressure (i.e.,  $\beta_{j \neq 1} = 0.008$ ). Type 1 shows the exact same large negative rate of change, while the remaining types follow the exact same positive linear relationship. The estimated slope of the linear relationship  $\beta_{j \neq 1} = 0.0025$  for the  $j \neq 1$  types does not correspond to the actual strength of selection on each of those types. This result is not an error since, as required by Equation (11), the rate of change across the  $j \neq 1$  types, calculated as  $\sum_{j \neq 1} 0.0025 \text{VAR}[z_i^{(j)}] + 0.000051$ , yields  $\sum_{j \neq 1} \Delta z^{(j)} = 0.0005064$ , which is the compliment to the rate of change observed for Type 1  $\Delta z^{(1)} = -0.0005036$ . More generally, Equation (11) suggests that  $\beta_k \text{VAR}[z_i^{(k)}] = -\beta_{j \neq k} \sum_{j \neq k} \text{VAR}[z_i^{(j)}]$  when  $\beta_{j \neq k}$  is a constant across the  $j \neq k$  types. If the summed variance across the  $j \neq k$  types is less than the variance exhibited by type  $k$ , then the magnitude of  $\beta_{j \neq k}$  will be greater than the magnitude of  $\beta_k$  to compensate. Conversely, if the summed variance across the  $j \neq k$  types is greater than the variance exhibited by type  $k$ , then the magnitude of  $\beta_{j \neq k}$  will be less than the magnitude of  $\beta_k$ . Only where the summed variance across the  $j \neq k$  types is exactly equal to the variance exhibited by type  $k$  will the magnitude of  $\beta_{j \neq k}$  be identical to that of  $\beta_k$ .

The pattern exhibited in Fig. 8a is not a mechanistic effect driven by the constraint that the frequencies rim types must sum to one within each ceramic ware. Rather, where one component of the assemblage—a single type or set of different types—is under selection, while the other component is not, the system will be organized around two relative payoff levels; for example,  $w_i^{(k)}/w$  for type  $k$  and  $w_0/w$  for the remaining  $j \neq k$  types. The relative proportions  $p_i$  of each ceramic ware  $i$  will change according to Equation (2) based on these two relative payoff levels. We have established that the pattern seen in Fig. 8a also emerges when multiple types within an assemblage share the same or a statistically related non-zero selection coefficient (i.e.,  $\beta_{j \neq k} \neq 0$ ). For example, the same qualitative outcome arises if seven of the simulated types have selection coefficients drawn from a probability distribution with positive mean and small standard deviation, while the one remaining type has a large negative selection coefficient. In this case, however, the quantitative estimates for the strengths of selection will differ from the true values because of covariance between types (see below). Importantly, the results in Fig. 8a presents a problem of equifinality. We are unable to establish observationally whether the pattern seen in Fig. 8a is the result of selection only on the single declining type, selection only on the seven types excluding Type 1, or two non-zero selection strengths operating on both components. In the case where one component is under selection while the other is not, the slope of the line connecting the single isolated point to the origin offers an accurate estimate of the true value of  $\beta$ . We favor the simplest of these hypotheses, where selection is operating primarily on one type, while the other types are all subject to no selection at all.

Fig. 8b models an analogous stochastic situation using the same initial ceramic assemblage in Fig. 8a. In this case, Type 1 undergoes stochastic fluctuations in frequency, while the remaining types do not experience stochastic fluctuations. Specifically,  $\delta_i^{(1)}$  is drawn from a Gaussian normal distribution with mean  $\mu = -0.008$  and standard deviation  $\sigma = 0.0001$ .

According to the Price Equation  $E[\delta_i^{(1)}] = -0.008$ , and we should expect Type 1 to show a negative rate of change, which is exactly what we see in Fig. 8b. However, it is not possible to estimate accurately the true value of  $E[\delta_i^{(1)}]$  since, in a single stratigraphic interval, we only have one realization of the random variable  $\delta_i^{(1)}$ . More stratigraphic intervals are needed to accurately estimate the mean  $E[\delta_i^{(1)}] = \mu$ . The observed negative rate of change in Fig. 8b may suggest that  $E[\delta_i^{(1)}] < 0$ , but this conclusion is largely dependent upon assuming that the standard deviation of the underlying stochastic distribution is small.

Despite the above limitation, it is still possible to characterize qualitatively the pattern in Fig. 8b as being driven by stochastic forces. Unlike in Fig. 8a, the rim types excluding Type 1 do not follow a linear increasing pattern. Rather, they exhibit no correlation with variance. Moreover, the equivalence of outcomes seen where selection is operating alternatively on one or the other component of the assemblage does not appear to hold in this case. If the  $j \neq 1$  types are all subjected to stochastic fluctuations drawn from a Gaussian normal distribution with mean  $\mu = 0.008$  (note the sign of  $\mu$ ) and standard deviation  $\sigma = 0.0001$ , while Type 1 does not fluctuate stochastically, we do not arrive at the outcome seen in Fig. 8b. Overall, the result presented in Fig. 8b is similar to a pure stochastic system where there should be no relationship between variance and rate of change in mean frequencies. The large negative rate of change for Type 1 is the only apparent deviation from simple stochasticity. Fig. 8c, by contrast, illustrates a completely neutral case where, starting with the same initial simulated assemblage, the frequency of each rim type changes by  $\delta_i^{(k)}$  drawn from a Gaussian normal distribution with mean  $\mu = 0.0$  and standard deviation  $\sigma = 0.0001$  (i.e.,  $E[\delta_i^{(k)}] = 0.0$ ). As expected, there is no consistent relationship between variance and rate of change in the mean frequency of rim types.

We may now be in a better position to interpret the results from Boğazköy-Hattusa. Fig. 7a presents uncorrected regression results for  $\Delta z$  on  $\text{VAR}[z_i]$ , for the seven types excluding Type I3. The slope of this relationship is an estimate of  $\hat{\beta}_{j \neq 3}$  at a time scale of 50 years, the assumed age difference between Oberstadt 3 and 2 (Steele et al., 2010). Fig. 7b uses Equation (9), with  $\tau/\Delta t = 1/50$ , to derive an estimate of  $\hat{\beta}_{j \neq 3}$  at an annual time scale. Focusing on the corrected results, simple linear regression applied to the seven types, excluding Type I3, yields a slope of  $\hat{\beta}_{j \neq 3} = 2.9345$  and intercept  $\hat{E}[\delta_i] = 0.0000112$ . The intercept is not significantly different from zero ( $t = 0.019$ ,  $p$ -value = 0.986) (Table 5). The estimated slope is not significant at  $\alpha = 0.05$  ( $t = 2.35$ ,  $p$ -value = 0.0654). However, at this early stage of method development it seems extreme to reject the hypothesis that the slope is different from zero. The slope of the line connecting Type I3 to the origin is  $\hat{\beta}_3 = -1.496$ . The requirement of Equation (11) is met by regression model presented in Fig. 7b and Table 5, providing a test the internal consistency of the numbers. Calculating  $\Delta z^{(j)} = 2.935 \text{VAR}[z_i^{(j)}] + 0.0000112$  for each recorded variance in rim types seven types, excluding Type I3, and summing the predicted values produces  $\sum_{j \neq 3} \Delta z^{(j)} = 0.00688$ , which is exactly the compliment of  $\Delta z^{(3)} = -0.00688$  found for Type I3 (see Table 4). Following the results in Fig. 8a, we suggest that the magnitude of  $\hat{\beta}_{j \neq 3} = 2.9345$  is probably an overestimate of the true selection coefficient since  $\sum_{j \neq 3} \text{VAR}[z_i^{(j)}] < \text{VAR}[z_i^{(3)}]$  (see Table 4) and consequently the magnitude of  $\hat{\beta}_{j \neq 3}$  must be greater than that of  $\beta_3$  to compensate.

The pattern of frequency changes in bowl rim types displayed at the Boğazköy-Hattusa is arguably most similar to that seen in Fig. 8a. The frequency of Type I3 rims show high variance in the Oberstadt 3 assemblage and a relatively large, negative rate of change. The frequencies of the remaining seven rim types suggest a single linear relationship. The distribution of the seven types, excluding Type I3, is not “flat” as might be expected if change in

**Table 5**  
Coefficients from regressions of change in the mean frequency of rim types on the variance in rim type frequencies for seven of eight rim types at Boğazköy-Hattusa.

	Time-scale	Coefficients	Standard Error	<i>t</i>	<i>p</i> -value	Lower 95%	Upper 95%
$E[\hat{\delta}_i]$ (intercept)	Uncorrected	0.000560	0.030031	0.018639	0.985850	−0.076636	0.077756
$\hat{\beta}_{j \neq 3}$ (slope)	Uncorrected	146.7232	62.3882	2.3518	0.0654	−13.6507	307.0971
$\hat{\beta}_3$ (slope) <sup>a</sup>	Uncorrected	−72.4810	—	—	—	—	—
$E[\hat{\delta}_i]$ (intercept)	Corrected	0.0000112	0.000601	0.018639	0.985850	−0.001533	0.001555117
$\hat{\beta}_{j \neq 3}$ (slope)	Corrected	2.9345	1.247763	2.3518	0.0654	−0.273014	6.141941
$\hat{\beta}_3$ (slope) <sup>a</sup>	Corrected	−1.4496	—	—	—	—	—

<sup>a</sup> Calculated as the slope of the line connecting the Type I3 point to the origin.

Type I3 was being driven solely by a negative stochastic process as in Fig. 8b. There also appears to be too much structure to assign the observed pattern entirely to a neutral stochastic, or “drift-like” process as in Fig. 8c. With caution, we conclude that selection is the dominant force operating on the Boğazköy-Hattusa ceramic bowl rim assemblage and that stochastic fluctuations play little or no role in driving change in the mean frequency of these rim types. This conclusion is broadly consistent with the results obtained by Steele et al. (2010) using different methods. It is unclear, however, whether selection at Boğazköy-Hattusa took the form of directional selection against Type I3 rims, with no selection on the remain rim types, or positive directional selection that was constant across seven rim types and no selection on Type I3. It is also possible that there were two different selective pressures operating at Boğazköy-Hattusa, one negative selective pressure impacting Type I3, and a different positive one impacting the remaining rim types. We favor the simplest hypothesis that selection was operating primarily against Type I3. We therefore emphasize the value  $\hat{\beta}_3 = -1.496$  in discussing the strength of selection below.

## 6. The strength of selection At Boğazköy-Hattusa

It is necessary to ask whether the estimate of  $\hat{\beta}_3$  (and  $\hat{\beta}_{j \neq 3}$ ) from Boğazköy-Hattusa represent strong or weak selection. This is fundamentally a comparative question. Unfortunately, we know of no studies that attempt to measure the strength of selection in the context of cultural transmission or cultural evolution. Several prominent meta-studies have been published regarding the strength of selection in the context of biological evolution (Endler, 1986; Hoekstra et al., 2001; Kingsolver et al., 2001). Hoekstra et al. (2001), for example, compare estimates of the absolute values of standardized selection gradients for 993 traits among 62 unique taxa. Standardized selection gradients are calculated as  $\beta'_j = \beta_j \sigma_j$  where  $\sigma_j$  is the standard deviation of the distribution of observed values  $z_j$  for trait  $j$ . Standardized selection gradients are therefore measured in units of standard deviation. Hoekstra et al. (2001) found that the absolute values of standardized selection gradients are negative exponentially distributed with a mean of  $|\beta'_j| = 0.153$  (for survival or viability selection)(see also Kingsolver et al., 2001). The value  $|\beta'_j|$  means that an individual with a trait value  $z_j$  one standard deviation above the mean will have a relative fitness of  $w_j/w = 1.153$ . Maximum standardized selection gradients were rarely seen to exceed  $|\beta'_j| = 0.5$ . Typically, selection in a biological context is considered to be “strong” if standardized selection gradients are  $|\beta'_j| > 1$  (Hoekstra et al., 2001).

At Boğazköy-Hattusa, we assume that the slope of the line  $\hat{\beta}_3 = -1.496$  calculated for Type I3 represents the best estimate for the selection coefficient at the site. The value for  $\hat{\beta}_{j \neq 3} = 2.9345$  is probably an overestimate of the strength of selection because the summed variance among the seven types from which this value is determined is less than the variance seen for Type I3 (see above). We convert  $\hat{\beta}_3$  to a standardized selection gradients by multiplying

this value with the standard deviation in frequency for Type I3 in Oberstadt 3 assemblage  $\sigma(z_i^{(3)}) = \sqrt{\text{VAR}[z_i^{(3)}]} = 0.06892$ . The standardized selection coefficients is then  $\hat{\beta}'_3 = -0.1031$  for Type I3. A ceramic ware with Type I3 sherd frequencies one standard deviation above the mean frequency  $z^{(3)}$  will have a relative fitness of  $w_i^{(3)}/w = 1 - 0.1031 = 0.897$ , which will lead to a rapid decline in the proportion of that ceramic ware in the assemblage. For  $\hat{\beta}_{j \neq 3}$ , the standardized selection gradient is calculated by multiplying this value with  $\sigma(z_i^{(j \neq 3)}) = \sqrt{\sum_{j \neq 3} \text{VAR}[z_i^{(j)}]} = 0.04816$ . The standardized selection coefficients is then  $\hat{\beta}'_{j \neq 3} = 0.1413$ . Both standardized selection gradients calculated for Boğazköy-Hattusa are surprisingly similar to the mean reported by Hoekstra et al. (2001) for biological evolution and neither may be said to constitute “strong” selection. The significance of this result will only be apparent as more attempts are made to calculate the strength of selection and stochastic forces from archaeological and other cultural contexts.

## 7. Discussion

The Price Equation is a flexible and practical tool for investigating the nature of evolutionary change in archaeological contexts. The flexibility of the Price Equation comes from its generality which, as demonstrated here, can accommodate the analysis of change in artifact frequencies from real archaeological contexts. The practical advantages of the Price Equation arise from the simple and intuitive connections it makes between model components and observable archaeological measures. Specifically, the Price Equation as developed here may be evaluated given data on the variance in frequencies and change in the mean frequencies of archaeological attributes. Such data are readily available from a diverse array of archaeological contexts.

The Price Equation also has theoretical appeal in that it is capable of detecting the simultaneous operation of selective and stochastic forces. Our approach thus differs from Steele et al. (2010), who primarily seek to identify deviations in the relative frequencies of rim types from a theoretical distribution expected at the mutation–drift equilibrium (see also Kimura, 1983; Kohler et al., 2004; Neiman, 1995). Where those deviations are statistically significant, Steele et al. (2010) point to selection as the likely culprit. The Price Equation reinforces the view that selection is indeed responsible for driving frequency changes in rim types at Boğazköy-Hattusa, and that stochastic forces played little or no role. This conclusion is drawn by regressing the change in the mean frequency of rim types ( $\Delta z^{(k)}$  in the notation used here) against the variance in rim type frequencies ( $\text{VAR}[z_i^{(k)}]$ ) for the two archaeological phases at the site. We find not only that rates of change are typically large when variance in frequency is high, which is a signature of selection in the absence of drift, but also that there is significant structure to the patterns of change seen in the assemblage. Specifically, the majority of rim types appear to be co-evolving in a directional pattern inconsistent with stochastic processes, including neutral, “drift-like” change. Co-evolution among these bowl rim types stands in contrast to the

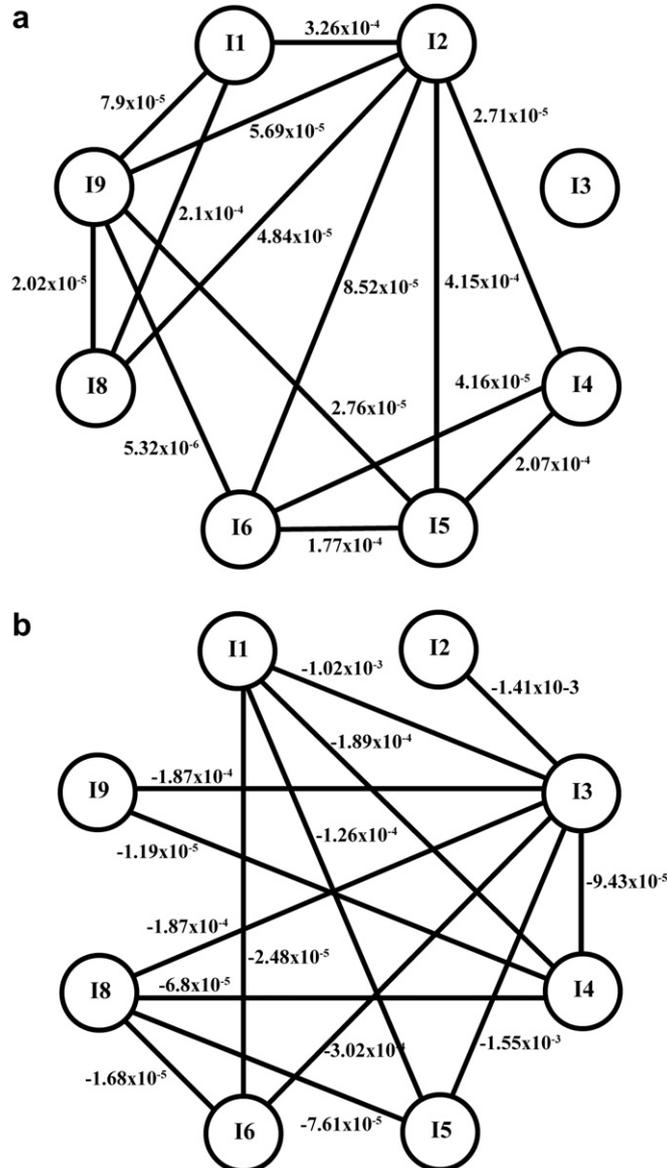
**Table 6**  
Variance-covariance matrix for the eight ceramic rim types from Boğazköy-Hattusa.

Type	I1	I2	I3	I4	I5	I6	I8	I9
I1	0.000745	0.000326	-0.001020	-0.000189	-0.000126	-0.000025	0.000210	0.000079
I2	0.000326	0.000451	-0.001410	0.000027	0.000415	0.000085	0.000048	0.000057
I3	-0.001020	-0.001410	0.004749	-0.000094	-0.001547	-0.000302	-0.000187	-0.000187
I4	-0.000189	0.000027	-0.000094	0.000088	0.000207	0.000042	-0.000068	-0.000012
I5	-0.000126	0.000415	-0.001547	0.000207	0.000922	0.000177	-0.000076	0.000028
I6	-0.000025	0.000085	-0.000302	0.000042	0.000177	0.000035	-0.000017	0.000005
I8	0.000210	0.000048	-0.000187	-0.000068	-0.000076	-0.000017	0.000069	0.000020
I9	0.000079	0.000057	-0.000187	-0.000012	0.000028	0.000005	0.000020	0.000010

frequency changes in Type I3 which, as recognized by Steele et al. (2010), decreases in relative frequency by more than a third between the Oberstadt 3 and 2 phases. The data are consistent with a conclusion that selection within the assemblage may have primarily operated against rim Type I3.

The Boğazköy-Hattusa assemblage also raises several intriguing questions about the nature of change in frequency among multiple related artifact types. Analysis of the Price

Equation indicates that the rate of change in the mean frequency of one rim type must be balanced by the cumulative rate of change among all other rim types. This is true for any focal rim type shown in Table 4. What is not immediately apparent is how change should be distributed among the *k* rim types in an assemblage. At Boğazköy-Hattusa we observed that Type I3 sherds with everted rims display a relatively large, negative rate of change in mean frequency and that this pattern of change is an



**Fig. 9.** Positive and negative covariance path diagrams for the Oberstadt 3 (older) phase at Boğazköy-Hattusa. (a) Positive covariance connections between ceramic rim types. (b) Negative covariance connections between ceramic rim types. Links are symmetrical. Covariance values are indicated adjacent to the line (see Table 6).

outlier within the assemblage as a whole. The change in the mean frequency of Type I3 bowl sherds is balanced, however, by change across the remaining types, which includes four types that show limited amounts of change in both positive and negative directions, and three types that show more significant frequency changes in the positive direction. Our controlled simulations offer some basis for inferring why change is distributed across multiple types in the ways seen at Boğazköy-Hattusa (see Fig. 8). However, we readily acknowledge that we have explored only a small number of the possible combinations of selective and stochastic parameters that could be operating at Boğazköy-Hattusa. Indeed, it is conceivable that each of the eight rim type could have its own unique selection coefficient  $\beta_k$  as well as stochastic fluctuations derived from unique underlying probability distributions, which need not be normal (Brantingham, 2007). Clearly, the observations above raise more questions than they answer, but they are nonetheless important to make.

Our approach to analysis of the Boğazköy-Hattusa assemblage has emphasized a version of the Price Equation developed for analysis of the evolution of single traits. As such, it is conceptually oriented towards detecting direct evolutionary effects; i.e., the effects of selection and stochastic forces operating directly on individual archaeological traits. Our analysis of multiple bowl rim types proceeded as if only simple direct effects operated on multiple types. While simulations seem to bear this approach out, it almost certainly will be necessary to develop appropriate procedures for dealing with both *direct* and *indirect* evolutionary effects within assemblages. In fact, an existing statistical architecture should be useful in this context. It has been shown that the total rate of change in a trait  $\Delta z^{(k)}$  given changes in correlated traits is (Lande and Arnold, 1983; Rice, 2004)

$$\Delta z^{(k)} = \beta_k \text{VAR}[z_i^{(k)}] + \sum_{j \neq k} \beta_{kij} \text{COV}[z_i^{(k)}, z_i^{(j)}] \quad (13)$$

where  $\beta_k$  is the regression coefficient of relative payoffs  $w_i^{(k)}/w$  on the frequency  $z_i^{(k)}$  of rim type  $k$ , the parameter  $\beta_{kij}$  is the regression coefficient of relative payoffs  $w_i^{(k)}/w$  accruing to rim type  $k$  given a frequency of rim type  $j$ , and  $\text{COV}[z_i^{(k)}, z_i^{(j)}]$  is the statistical covariance between the frequency of rim type  $k$  and rim type  $j$ . The first term on the right hand side of Equation (13) reflects *direct* selection effects, while the second term reflects the sum of all *indirect* selection effects. Conceptually, the second term describes how selection for or against one rim type actually filters through a network of evolutionary dependencies to impact the frequency of other rim types. Additional independent terms would have to be added to Equation (13) to account for direct and indirect stochastic effects to be consistent with the version of the Price Equation deployed here.

The implications of Equation (13) are very broad and, frankly, very complicated. A complete analysis of the Boğazköy-Hattusa assemblage in terms of Equation (13) is therefore beyond the scope of this paper. It is possible to get a feeling for how complicated indirect selective effects may be by examining the covariance structures within the assemblage. Table 6 shows the complete variance-covariance matrix for the Obserstadt 3 phase ceramics at Boğazköy-Hattusa. Fig. 9 shows graphically the positive and negative covariance patterns among rim sherd frequencies. The patterns are surprisingly complex. Regarding patterns of positive covariance, where two types are simultaneously above or below the mean frequency, Type I2 bowls with simple thickened rims and Type I9 bowls with inverting walls are highly connected (Fig. 9a). This suggests that selection operating on these types will filter through to have indirect positive effects on many other types. By contrast, Type I3 bowls with everted rims do not positively covary with any other types, suggesting

that negative selection operating against Type I3 rims cannot indirectly drive *negative* change among other types. This last observation is complimented by examining negative covariances within the assemblage. In Fig. 9b, Type I3 negatively covaries with all other types, indicating that selective effects on Type I3 should have indirect selective effects on all other types in the opposite direction. Type I1, I4 and I8 negatively covary with four other types and are the next most likely to distribute opposing indirect effects within the assemblage. However, it should be noted that the strength of positive and negative covariance relationships varies over a wide range (Table 6). The largest absolute positive covariance (Types I2 and I5) is a factor of 78 greater than the smallest positive covariance (Types I6 and I9). The largest absolute negative covariance (Types I3 and I5) is a factor of 37 greater than the smallest negative covariance (Types I4 and I6). Indirect selective effects will thus be proportionally greater for those relationships that show greater covariance.

## 8. Conclusions

We have shown that selective and stochastic forces can be estimated using statistical procedures such as linear least-squares regression. Based on the Price Equation, this approach urges us to consider the statistical relationships between rate of change in mean attribute values and the variance in those values. The data published by Steele et al. (2010) on Late Bronze Age Hittite ceramic assemblages from Boğazköy-Hattusa, Turkey, are minimally sufficient to investigate the nature of frequency changes in bowl rim types across two archaeological phases. Our analysis confirms the broad conclusions offered by Steele et al. (2010) that payoff-correlated changes, or what we describe as selection, play a significant role in the observed changes in rim frequencies, and that stochastic evolutionary forces play little or no role.

Our numerical results describing selection as dominant force at Boğazköy-Hattusa come with several caveats, however. First, the absolute magnitudes we attribute to selective forces, do not include corrections that might have to be made for unknown changes in mean fitness (see Endler, 1986). Second, we have not explored all possible combinations of selective and stochastic forces and therefore cannot say with certainty some other combination of these parameters might explain the observed patterns of variance and rates of change equally well. Finally, acknowledging that there is a complex covariance structure among rim types at Boğazköy-Hattusa indicates that a complete model will necessarily take into account both direct and indirect selective effects as well as those of purely stochastic forces. Despite these caveats we believe that the Price Equation offers one way forward for testing models of cultural evolution in archaeological contexts.

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## References

- Brantingham, P.J., 2007. A Unified evolutionary model of archaeological style and function based on the Price equation. *American Antiquity* 42, 395–416.
- Brantingham, P.J., Surovell, T.A., Waguespack, N.M., 2007. Modeling post-depositional mixing of archaeological deposits. *Journal of Anthropological Archaeology* 26, 517–540.
- Boyd, R., Richerson, P.J., 1985. *Culture and the Evolutionary Process*. University of Chicago Press, Chicago.
- Eerkens, J.W., Lipo, C.P., 2005. Cultural transmission, copying errors, and the generation of variation in material culture and the archaeological record. *Journal of Anthropological Archaeology* 24, 316–334.
- Endler, J., 1986. *Natural Selection in the Wild*. Princeton Univ Press, Princeton.

- Ewens, W., 1972. The sampling theory of selectively neutral alleles. *Theoretical Population Biology* 3, 87–112.
- Frank, S.A., 1997. The Price Equation, Fisher's fundamental theorem, kin selection, and causal analysis. *Evolution* 51, 1712–1729.
- Frank, S.A., Slatkin, M., 1992. Fisher's fundamental theorem of natural selection. *Trends in Ecology & Evolution* 7, 92–95.
- Glatz, C., 2009. Empire as network: spheres of material interaction in late Bronze age Anatolia. *Journal of Anthropological Archaeology* 28, 127–141.
- Hoekstra, H.E., Hoekstra, J.M., Berrigan, D., Vignieri, S.N., Hoang, A., Hill, C.E., Beerli, P., Kingsolver, J.G., 2001. Strength and tempo of directional selection in the wild. *Proceedings of the National Academy of Sciences of the United States of America* 98, 9157–9160.
- Hubbell, S.P., 2001. *The Unified Neutral Theory of Biodiversity and Biogeography*. Princeton University Press, Princeton.
- Hunt, G., 2004. Phenotypic variation in fossil samples: modeling the consequences of time-averaging. *Paleobiology* 30, 426.
- Kimura, M., 1983. *The Neutral Theory of Molecular Evolution*. Cambridge University Press, Cambridge.
- Kingsolver, J., Hoekstra, H., Hoekstra, J., Berrigan, D., Vignieri, S., Hill, C., Hoang, A., Gibert, P., Beerli, P., 2001. The strength of phenotypic selection in natural populations. *American Naturalist* 157, 245–261.
- Kohler, T.A., VanBuskirk, S., Ruscavage-Barz, S., 2004. Vessels and villages: evidence for conformist transmission in early village aggregations on the Pajarito Plateau, New Mexico. *Journal of Anthropological Archaeology* 23, 100–118.
- Lande, R., Arnold, S.J., 1983. The measurement of selection on correlated characters. *Evolution* 37, 1210–1226.
- Lyman, R., 2003. The influence of time averaging and space averaging on the application of foraging theory in zooarchaeology. *Journal of Archaeological Science* 30, 595–610.
- Mielke, D., Schoop, U., Seeher, J., 2006. Strukturierung und Datierung in der hethitischen Archäologie: Voraussetzungen, Probleme, neue Ansätze. *Yayınları, Istanbul*.
- Neiman, F.D., 1995. Stylistic variation in evolutionary perspective: inferences from decorative diversity and interassemblage distance in Illinois Woodland ceramic assemblages. *American Antiquity* 60, 7–36.
- Perreault, C.P., n.d. The impact of site sample size on the reconstruction of culture histories. *American Antiquity*, in press.
- Price, G.R., 1970. Selection and covariance. *Nature* 227, 520–521.
- Rice, S.H., 2004. *Evolutionary Theory: Mathematical and Conceptual Foundations*. Sinauer, Sunderland, MA.
- Richerson, P., Boyd, R., 2005. *Not by genes alone: How culture transformed human evolution*. University of Chicago Press.
- Schoop, U.-D., 2006. Dating the Hittites with statistics: ten pottery assemblages from Boğazköy-Hattuša. In: Mielke, D.P., Schoop, U.-D., Seeher, J. (Eds.), *Strukturierung und Datierung in der Hethitischen Archäologie*. *Yayınları, Istanbul*.
- Seeher, J., 2006. Chronology in Hattuša: new approaches to an Old problem. In: Mielke, D.P., Schoop, U.-D., Seeher, J. (Eds.), *Strukturierung und Datierung in der Hethitischen Archäologie*. *Yayınları, Istanbul*.
- Slatkin, M., 1994. An exact test for neutrality based on the Ewens sampling distribution. *Genetical Research* 64, 71–74.
- Slatkin, M., 1996. A correction to the exact test based on the Ewens sampling distribution. *Genetical Research* 68, 259–260.
- Steele, J., Glatz, C., Kandler, A., 2010. Ceramic diversity, random copying, and tests for selectivity in ceramic production. *Journal of Archaeological Science* 37, 1348–1358.
- Surovell, T.A., Brantingham, P.J., 2007. A note on the use of temporal frequency distributions in studies of Prehistoric Demography. *Journal of Archaeological Science* 34, 1868–1877.
- Templeton, A., 2002. Out of Africa again and again. *Nature* 416, 45–51.
- Templeton, A.R., Routman, E., Phillips, C.A., 1995. Separating population structure from population history: a Cladistic analysis of the Geographical distribution of Mitochondrial DNA Haplotypes in the Tiger Salamander, *Ambystoma tigrinum*. *Genetics* 140, 767–782.